State-Space “Switching” Model of DC-DC Converters in SystemVerilog.

Elvis Shera, Dialog Semiconductor
Use Case: Boost Converter

Secondary effects:

- Parasitic resistors ($R_L$, $R_C$).
- Body diodes for the switches.

$V_{OUT} \geq V_{IN}$
Step 1: Replace Mosfet with simpler model

Discontinuities are created due to the ideal switches !!!

More options are available for modeling transistors...
Step 2: Linearize the resulting circuit

Create possible circuit configurations:

- Ideal switch positions
- Diode polarity,
Step 3: Write circuit equations

For each configuration:

1. Define its differential equations:
   \[ m \frac{d^2 x(t)}{dt^2} = F(x(t)) \]

2. Bring the equations in Vector notation via A,B,C,D matrices (SS Model)
   \[ \dot{x}(t) = Ax(t) + Bu(t) \]
   \[ y(t) = Cx(t) + Du(t) \]

![Circuit diagram]

1. \[ I_L = \frac{dI_L}{dt} = \frac{1}{L} V - \frac{R}{L} I_L + \frac{1}{L} V_C \]
   \[ V_C = \frac{dV_C}{dt} = \frac{1}{C} I_L \]

2. \[ \begin{bmatrix} I_L \\ V_C \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \frac{1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V \]
Step 4: Identify conditions for which we need to switch to a certain configuration

At any given time, the circuit can only be in one of the configurations therefore simulate that

- All configurations have in common the same state variables and their derivatives.
- Each configuration has its own matrix coefficients.
- @ each switching point, only the coefficient need to be replaced with those of the configuration we are going to as next. Replace the matrix at any switching point.
Step 5: Define the computation step

- The state variable and their derivatives must be recalculated at every time step $T$.
- The time step has to be decided based on the system requirements.
- Since in discrete time domain, the State Space equations should be discretized before, but if the time step is quite small the two systems will be converging.

```verilog
bit clk; initial forever #(T*1s) clk = !clk;
var real a00, a01, a10, a11;
var real b00, b01, b10, b11;

always @(clk) begin
  // calculate the first derivatives
  dx0 = a00*x0 + a01*x1 + b00*J + b01*Vin;
  dx1 = a10*x0 + a11*x1 + b10*J + b11*Vin;
  // calculate the next state variables
  x0 = x0 + dx0*T;
  x1 = x1 + dx1*T;
end
```
Differences with Average SS Model

Current in a boost converter. Continuous conduction mode (CCM).

• Averaging is ok as long as we know all the times and have fixed switching frequencies.

• Other mode of operation like pulse-skip or discontinuous mode of operation are difficult as we do not have a fixed $T$ anymore.

• Behaviours happening within the switching cycle are not possible to observe due to the averaging effect. Example is power dissipation.

• Switching model does not relay on any Period or Duty Cycle information.

Create one matrix from all the existing one

\[
A_{avg} = DA_a + (1-D)A_b
\]

\[
B_{avg} = DB_a + (1-D)B_b
\]
Simulation Results

- Startup from discharged output capacitor
- Inductor current
- Capacitor voltage

Average Model
Switching Model
In-Cycle Power Dissipation

Load current = 100mA
p2p coil current ripple = 0.861 mA
average current = 126 mA

p2p voltage ripple = 0.8 mV
average voltage = 3.403 V
switching Frequency = 1.5 MHz
Simulation Results – Modes of Operation

Discontinuous Mode

- Load current: 200mA
- Peak coil current: 565 mA
- Average current: 370 mA
- p2p voltage ripple: 0.5 mV
- Average voltage: 3.408 V
- Switching Frequency: 1.5 MHz

Pulse Skipping

- Load current: 100mA
- p2p voltage ripple: 12 mV
- Average voltage: 3.404 V
- Frequency: 45 KHz
Simulation Results - Transients

Overlay Switching and average SS model.
Transient Line: Vin = Vin + 0.3V
Max peak = 25 mV

Overlay Switching and average SS model.
Transient Load of 3.1A
Max dip = 125 mV
Caveats

• If the topology changes, we need to rewrite the equations.

• Complex circuits can have many configurations, each requiring to write its own equation.
Conclusions

• Switching SS model has been presented.

• Advantages over the commonly used “average” model have been shown.
  – extending the modelling capabilities to architectures and modes of operation which are difficult/not possible with the typical average model.
  – Observation of high order effects like efficiency, power dissipation, etc...
  – Possibility of simulating all together with the digital controller, within one environment.

• Switching model is not slower than an “average” model as the switching is at much higher time step with respect to the computation step (> 2° orders)
Questions

Finalize slide set with questions slide