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Performance Evaluation of a Phase-Locked Loop using Variation-Aware Behavioral Models

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Problem Statement

AMS Verification:
Requires extensive simulation due to **p**rocess, **v**oltage, and **t**emperature **(PVT)** variations. Alternately, there is a substantial increase in number of process parameters – growth of ~20% every 4 to 5 yrs[1].

Need:
Accurate, efficient propagation of circuit-level PVT variations in system analysis.

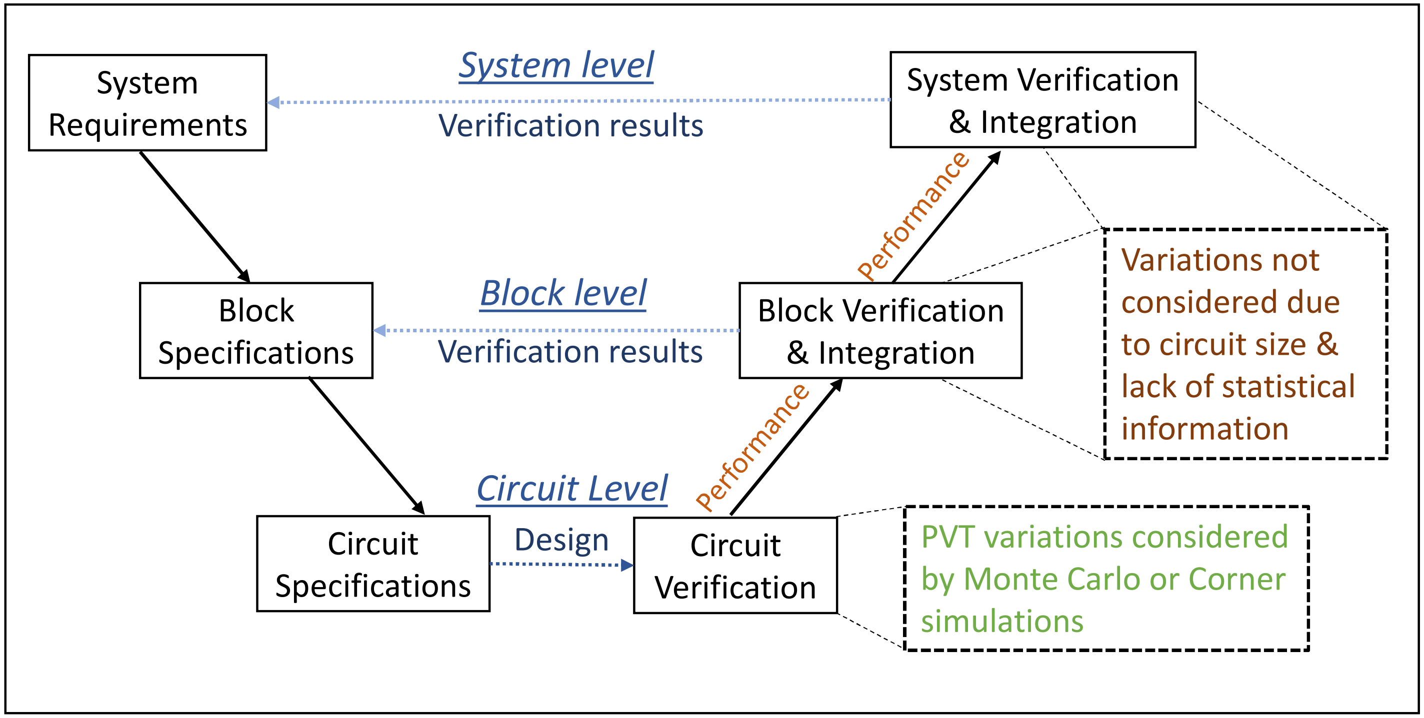

Fig: V-model for hardware development

[1] X. Li, J. Le, and L. T. Pileggi, "Statistical performance modeling and optimization," Foundations and trends in Electronic Design Automation, vol. 1, no. 4, pp. 331–480, 2006, ISSN: 1551-3939, 1551-3947. DOI: 10.1561/1000000008.

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- State-of-the-Art
- Proposed Variation-Aware Behavioral Models (VABMs)
- Hierarchical Framework
- Evaluation: Phase-Locked Loop
- Summary and Outlook

State-of-the-Art

State-of-the-Art

Statistical behavioral modeling considering PVT variations:

Polynomial-based approaches

- Response Surface Modeling (RSM) - Felt et al.[2], Filiol et al.[3]
- Random Polynomials - Lange et al.[4]
- Universal Kriging - Okobiah et al.[5]

+ High model accuracy using quadratic models
+ Parameter correlations using matrix($N \times N$)
- Model complexity ~$O(N^2)$

Learning-based approaches

- Correlated Bayesian Model Fusion (BMF) using Bayesian inference - Gao et al.[6], Alawieh et al.[7]
- Particle Swarm Optimization (PSO) - Garitselov et al.[8]

- Increased fitting complexity and training cost
- No capture of parameter correlations

Linear model-based approaches

- Quasi-Sensitivity Analysis - Kuo et al.[9]
- Affine Kernel Approach - Zivkovic et al.[10]

+ Accuracy comparable to RSM with reduced computational costs
+ Correlation preserved by models; No need for correlation matrix

Our behavioral models are constructed using Affine Kernel approach.

#N: Number of PVT parameters/ Number of random variables

Variation-Aware Behavioral Models

Hierarchical Approach

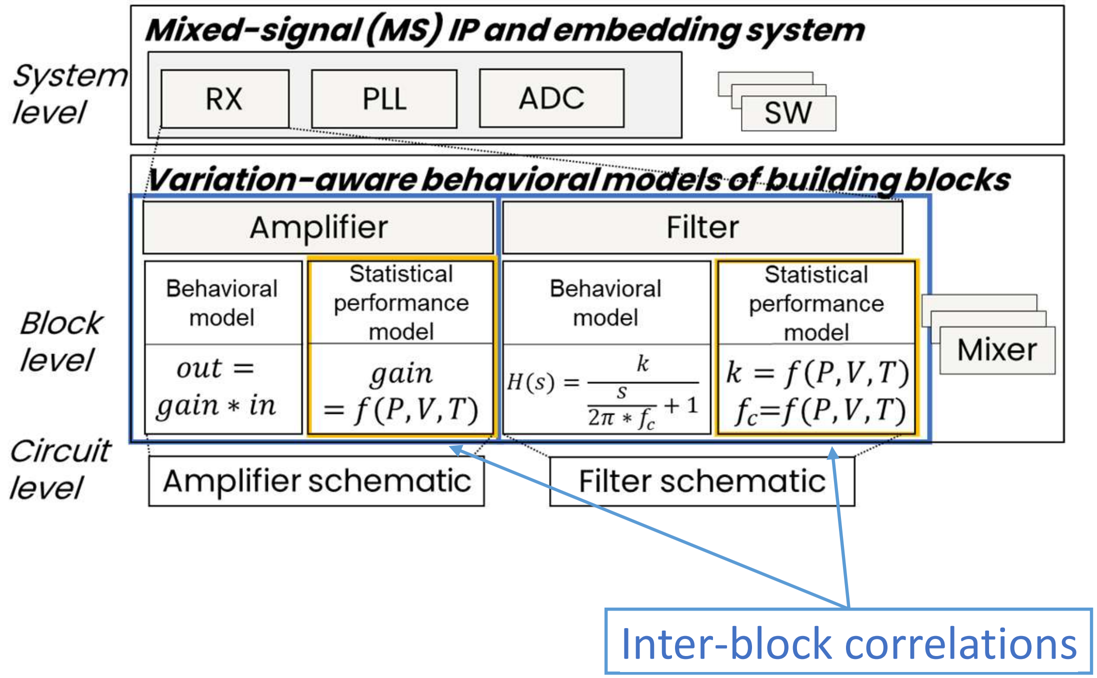
1. Statistical Performance Models(SPM)

➡ Model circuit performances as a function of PVT variations

2. Variation-Aware Behavioral Models (VABM)

➡ Combine functional behavioral models with SPMs

➡ Account inter-block parameter correlations



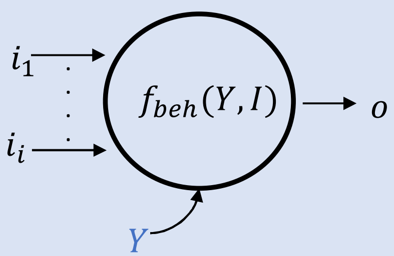
Variation-Aware Behavioral Model (VABM)

FUNCTIONAL BEHAVIORAL MODEL

- Behavior defined as a function:

$$f_{beh}: Inputs \rightarrow Outputs$$

- Parameters (Y) <=> key performance metrics of the modelled circuit block



(STATISTICAL) PERFORMANCE MODEL

- Model performance metrics as functions of P,V,T variations

$$y_j \sim f(P, V, T), \qquad y_j \in Y$$

$$y_j \sim y_{nom} + \sum_{k=1}^{n} \delta_k s_k$$

y_{nom} – nominal value of performance y_j

δ_k – variation of k^{th} parameter

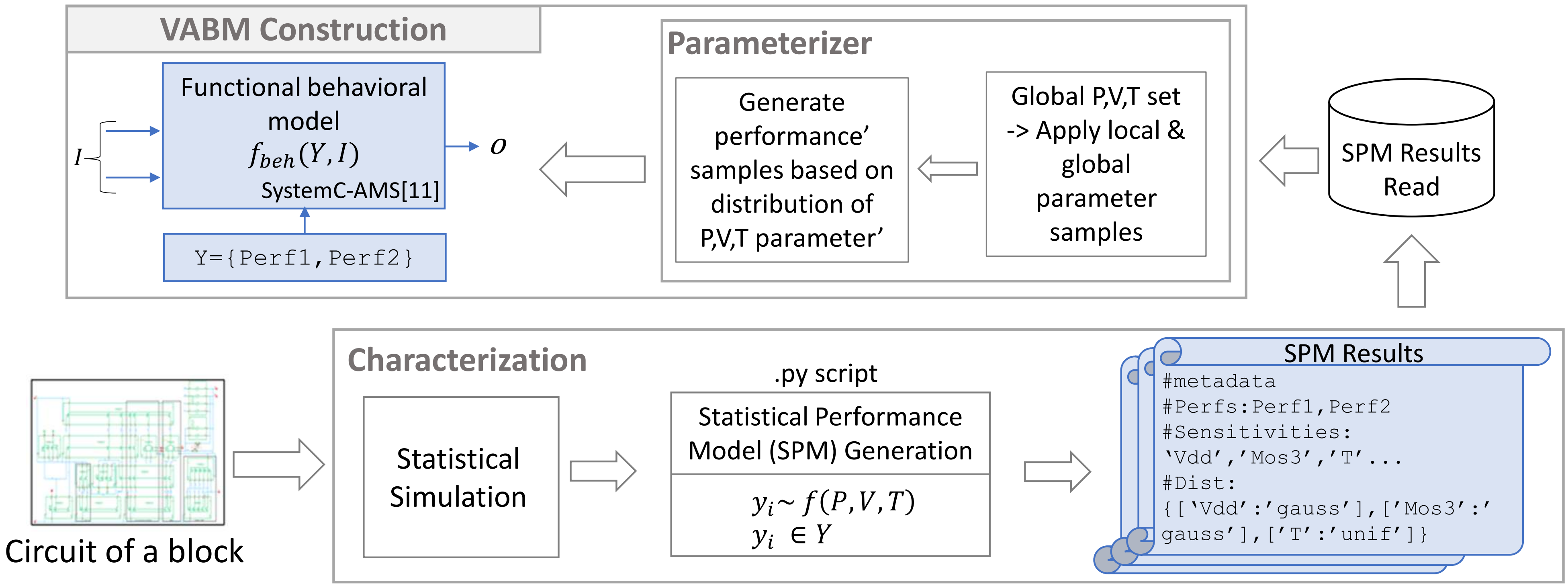
s_k – sensitivity coefficient

- Model generated from statistical simulation data

VARIATION-AWARE BEHAVIORAL MODEL

- Parameters sampled based on distributions estimated by the model

Hierarchical Framework: Bringing characterization data to behavioral block-level models



[11] "IEEE Standard for Standard SystemC(R) Analog/Mixed-Signal Extensions Language Reference Manual," Jan. 2016, doi: https://doi.org/10.1109/ieeestd.2016.7448795.

Evaluation

Phase-Locked Loop

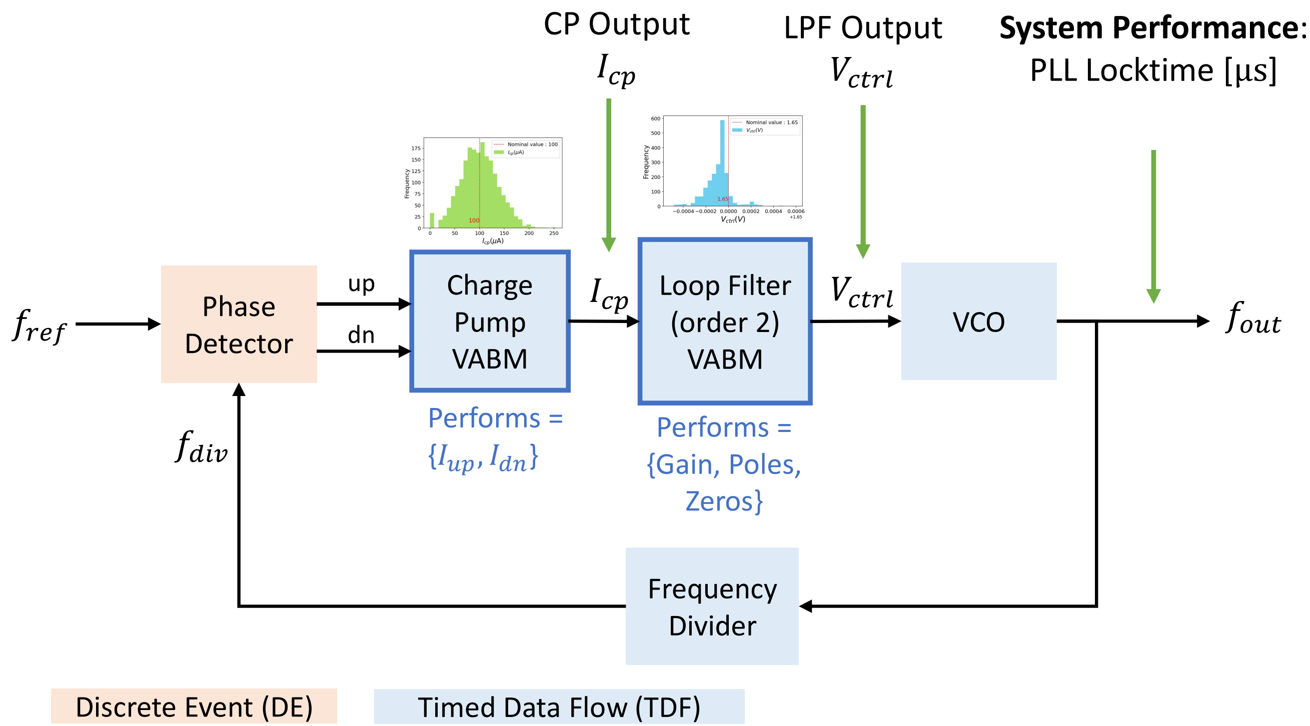


Fig: SystemC AMS model of a phase-locked loop[12]

VABMs act as plug & play models in hierarchical analysis.

VABMs takes care for inter-block correlations.

Block Performance Metrics	No. of PVT parameters
I_{up}, I_{dn}	35
$Gain, Poles, Zeros$	53
Shared variations CP and LPF	23

Model Accuracy: VABM vs Circuit Monte Carlo

Feature	
Monte Carlo Runs	2000
Probability Distribution Function (PDF) Comparison	Jenson Shannon Divergence (JSD)[13]
Statistical Measures	Mean(μ), Standard Deviation(σ), Skewness(γ), Kurtosis(β_2)
Target Sigma	±3σ

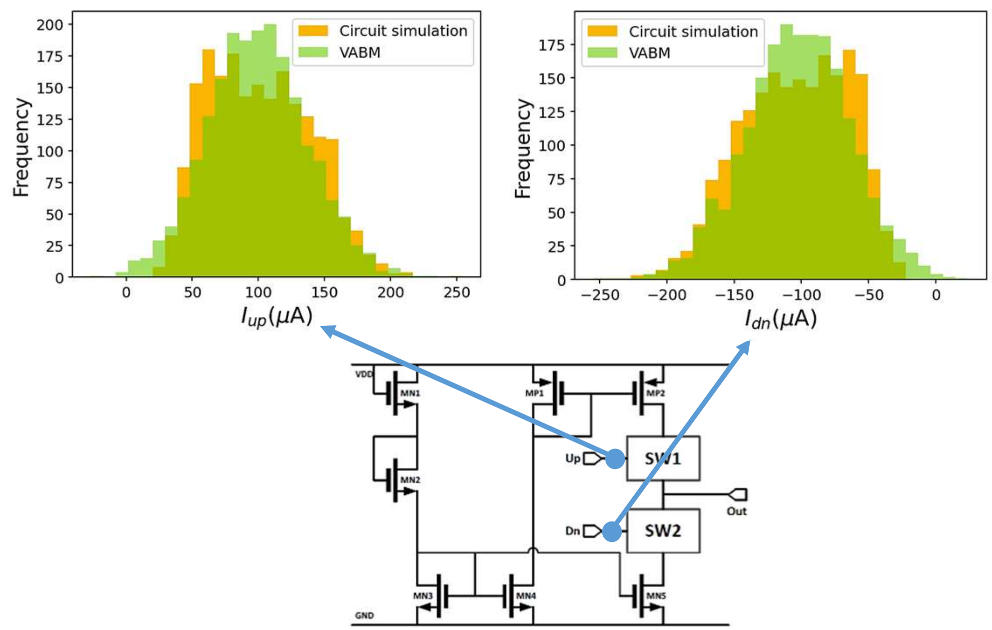


Fig: Example Charge Pump

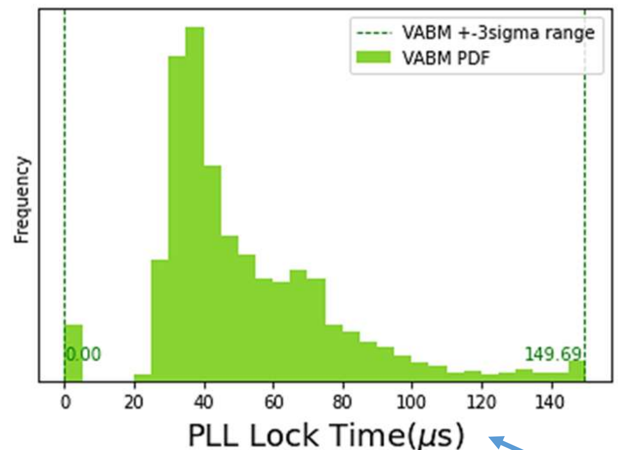
[13] J. Lin, "Divergence measures based on the shannon entropy," IEEE Transactions on Information Theory, vol. 37, no. 1, pp. 145–151, 1991. DOI: 10.1109/18.61115

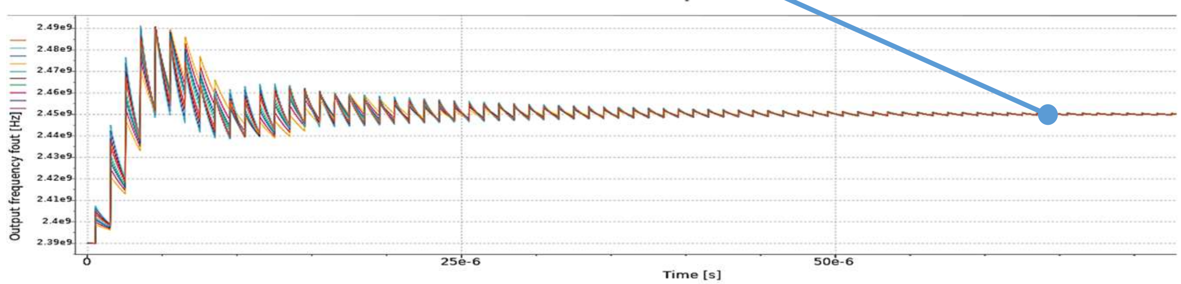
System Analysis

PLL System Performance using VABMs

- Performance metric: Lock time of PLL
- PDFs also highlight cases where PLL fails to lock (at 0 seconds)
- #Monte Carlo runs: 2000

Simulation Metrics	Value
Single run time	0.76 [s]
Simulation overhead incl. Parameterizer	~ 0.03 [s]
Total wall clock time	$2000 \times 0.76[s]$ ~ 25 mins





Example: PLL output frequency for 10 Monte Carlo runs
Model simulated for 200 [µs], Sampling rate 100 [*ps*]

System Analysis: PLL

- Charge Pump Variation – PLL locally sensitive to CP
- Loop Filter Variation – Poor damping introduces harmonics in locking behaviour
- CP + LPF Combined Variation
  - Capture effect of inter-block correlations on PLL
  - Charge pump removes meta-stability

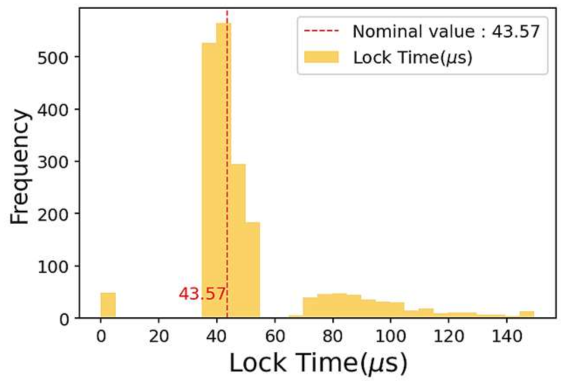


Fig: Charge pump variation effect

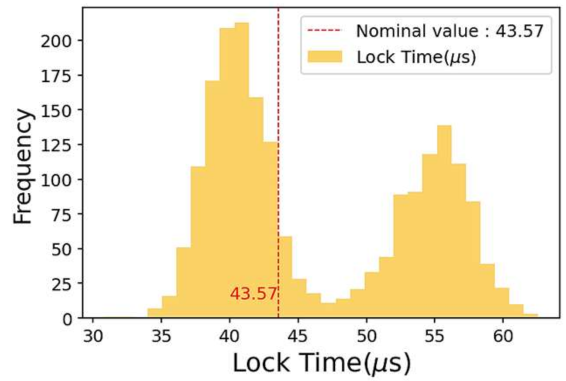


Fig: Loop filter variation effect

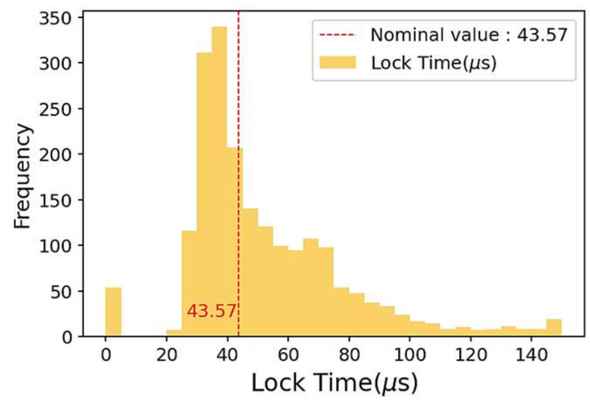


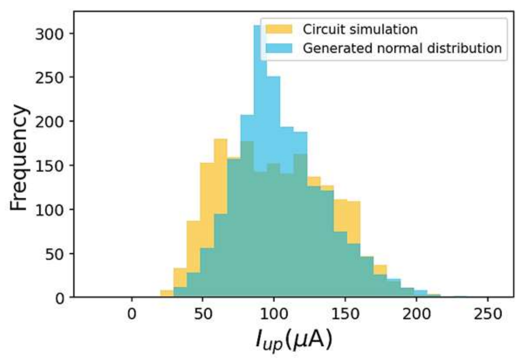
Fig: CP + LPF variation effect

Comparison with other scenarios

Gaussian Model

What if P,V,T variations and dependencies are not considered in system level?
Solution -> All block performance metrics are independent random variables.

Scenario 1:
Gaussian model – Assume block performance metrics follow a normal distribution.



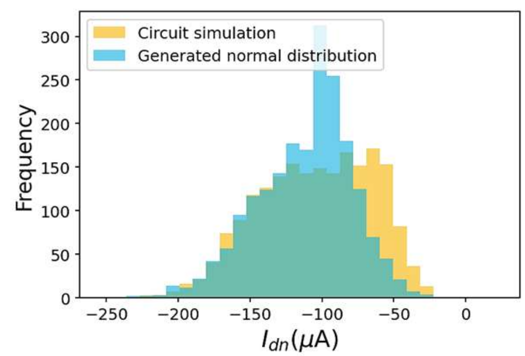


Fig: Probability distribution functions(PDFs) of charge pump currents – Gaussian model vs Circuit simulation

Gaussian Model: Limitations

Limitations:

- ❑ Assumes symmetric distribution
- ❑ No information on PVT data & inter-block correlations
- ❑ Estimated ±3σ ranges are narrow, potentially overlooking outlier behavior

FMKL Generalized Lambda Distribution (FMKL GLD) model

Scenario 2: <u>FMKL Model</u>: Freimer, Mudholkar, Kollia, and Liontas [14]

- Circuit performance distributions modeled using FMKL GLD.
- Quantile function defined by inverse cumulative distribution [14,15]

$$F^{-1}(u) = \lambda_1 + \frac{\frac{u^{\lambda_3} - 1}{\lambda_3} - \frac{(1-u)^{\lambda_4} - 1}{\lambda_4}}{\lambda_2}$$

$$\text{where } 0 < u < 1$$

Parameters:

λ_1 $- location$
λ_2 $- scale$
$\lambda_3, \lambda_4 - shape$

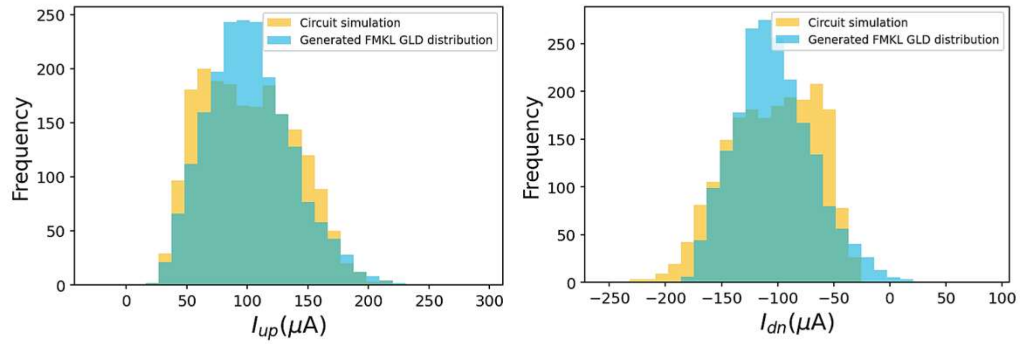
[14] A. Lange, I. Harasymiv, O. Eisenberger, F. Roger, J. Haase, and R. Minixhofer, “Towards probabilistic analog behavioral modeling,” in 2015 IEEE International Symposium on Circuits and Systems (ISCAS), Lisbon, Portugal: IEEE, May 2015, pp. 2728–2731, ISBN: 978-1-4799-8391-9. DOI: 10 . 1109 /ISCAS.2015.7169250.

[15] Z. A. Karian and E. J. Dudewicz, “Fitting the generalized lambda distribution to data: A method based on percentiles,” Communications in Statistics - Simulation and Computation, vol. 28, no. 3, pp. 793–819, 1999. DOI: 10 . 1080 /03610919908813579. eprint: https : / / doi . org / 10 . 1080 /03610919908813579.

FMKL GLD Model: Limitations

Limitations:

- ❑ No information on PVT data & inter-block correlations
- ❑ Limited support for $0 \leq \lambda_3$, λ_4[16]
- ❑ No closed-form for PDF; values derived from the quantile function



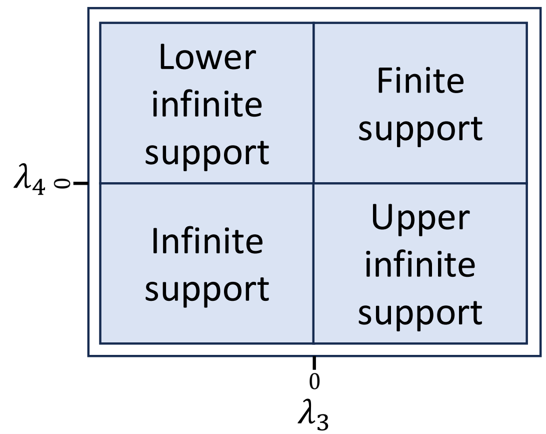


Fig: FMKL GLD Support Region [16]

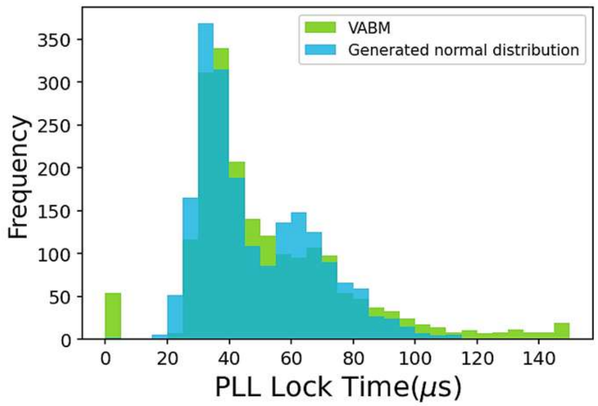
[16] Y. Chalabi, D. J. Scott, and D. Wuertz, “Flexible distribution modeling with the generalized lambda distribution,” 2012.

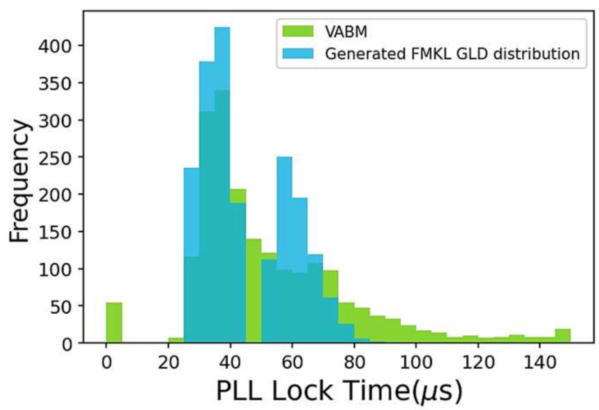
Effect of inter-block correlation

- Lower variances in PLL spread for FMKL GLD and Gaussian model.
- VABM spread similar to Gaussian model but also accounts effect of inter-block correlations.

Performance	Metrics	VABM	Gaussian Model	FMKL GLD -based model
Locktime [μs]	Mean	44.35	41.55	39.50
	Std. Dev.	22.29	21.40	18.54
	Skewness	1.28	0.91	0.59
	Kurtosis	3.22	2.32	1.88
	JSD	-	0.0438	0.0433
	$\pm 3\sigma$ range	[0.00,149.69]	[17.60,121.62]	[25.55,86.63]

Fig: Comparison of system performance metrics using all approaches





Summary

Summary

- A scalable hierarchical methodology for propagating process variations to system-level simulations using VABMs
- Effectively account for inter-block parameter correlations
- Effectively capture circuit behavior while aligning boundaries of the distribution
- Accelerate simulation speedup with minimal computational overhead

Outlook

- System analysis using improved SPM (employing piecewise linear models) over linear affine kernel.
- Explore significance of number of P,V,T parameters on system performance.
- Evaluate system performance using hierarchical regression analysis.
- Explore advanced statistical sampling techniques to observe tail behavior.

Thank you!
Questions?

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State-of-the-art references

[2] E. Felt, S. Zanella, C. Guardiani, and A. Sangiovanni-Vincentelli, “Hierarchical statistical characterization of mixed-signal circuits using behavioral modeling,” in Proceedings of International Conference on Computer Aided Design, Nov.1996, pp. 374–380. DOI: 10.1109/ICCAD.1996.569824
[3] H. Filiol, I. O’Connor, and D. Morche, “A new approach for variabilityanalysis of analog ICs,” in Proc. 2009 Joint IEEE North-East Workshop on Circuits and Systems and TAISA Conference (NEWCAS-TAISA), 2009.DOI: 10.1109/NEWCAS.2009.5290422.
[4] A. Lange, I. Harasymiv, O. Eisenberger, F. Roger, J. Haase, and R. Minixhofer, ‘Towards probabilistic analog behavioral modeling’, in 2015 IEEE International Symposium on Circuits and Systems (ISCAS), 2015, pp. 2728–2731.
[5] O. Okobiah, S. P. Mohanty, and E. Kougianos, “Fast statistical process variation analysis using universal kriging metamodeling: A pll example,” in 2013 IEEE 56th International Midwest Symposium on Circuits and Systems (MWSCAS), 2013, pp. 277–280. DOI: 10.1109/MWSCAS.2013.6674639

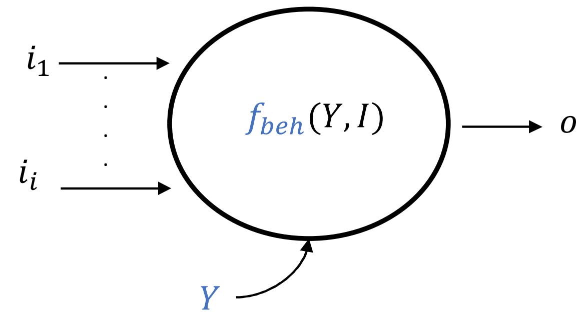
[6] Z. Gao, F. Wang, J. Tao, Y. Su, X. Zeng, and X. Li, “Correlated bayesian model fusion: Efficient high-dimensional performance modeling of analog/RF integrated circuits over multiple corners,” IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 42, no. 2, pp. 360–370, Feb. 2023, ISSN: 1937-4151. DOI: 10 . 1109/TCAD.2022.3174170.
[7] M. B. Alawieh, F. Wang, and X. Li, “Efficient hierarchical performance modeling for analog and mixed-signal circuits via bayesian co-learning,” IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 37, no. 12, pp. 2986–2998, Dec. 2018, ISSN: 1937-4151. DOI:10.1109/TCAD.2018.2789778
[8] O. Garitselov, S. P. Mohanty, E. Kougianos, and G. Zheng, “Particle swarm optimization over non-polynomial metamodels for fast process variation resilient design of nano-cmos PLL,” in Great Lakes Symposium on VLSI, Salt Lake City, UT, USA, ACM, 2012, pp. 255–258. DOI: 10.1145/2206781.2206843

[9] C.-C. Kuo, M.-J. Lee, C.-N. Liu, and C.-J. Huang, ‘Fast Statistical Analysis of Process Variation Effects Using Accurate PLL Behavioral Models’, IEEE Transactions on Circuits and Systems I: Regular Papers, vol. 56, no. 6, pp. 1160–1172, Jun. 2009
[10] C. Zivkovic, J. Roedel, N. Chavan, F. Rethmeier, and C. Grimm, ‘Variation-Aware Performance Verification of Analog Mixed-Signal Systems’, in DVCon Europe 2023; Design and Verification Conference and Exhibition Europe, 2023, pp. 7–13.

Proposed VABM

- Our proposed VABMs are defined based on the concept of parameter adaptive models[2].
- The functional process (f_{beh}) represents functional behavior
- (Y) enables functional adaptivity using Affine kernel[3]

$$\boldsymbol{f_{beh}: I \times Y \rightarrow O}$$



where

$I = \{i_1, i_2, \dots i_i\}; I\ is\ set\ of\ input\ signals$

$o \in O, where\ O\ is\ a\ set\ of\ output\ signals$

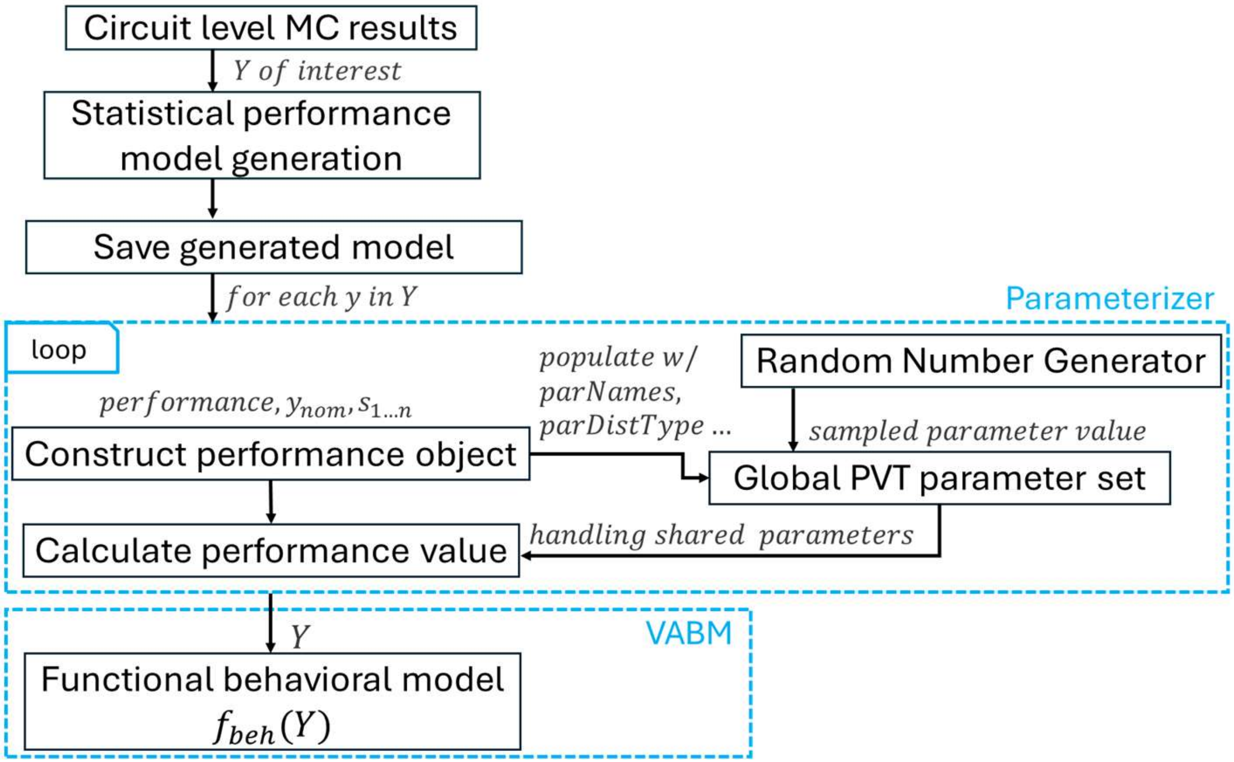
$Y = \{y_j | y_j \in \mathbb{R}, 1 \leq j \leq m\}; m\ are\ performance\ metrics$

[2] I. Sander and A. Jantsch, “Modelling adaptive systems in ForSyDe,” Electronic Notes in Theoretical Computer Science, Proceedings of the First Workshop on Verification of Adaptive Systems (VerAS 2007), vol. 200, no. 2, pp. 39–54, Feb. 29, 2008, ISSN: 1571-0661. DOI: 10.1016/j.entcs.2008.02.011

[3] C. Zivkovic, J. Roedel, N. Chavan, F. Rethmeier, and C. Grimm, “Variation-aware performance verification of analog mixed-signal systems,” in DVCon Europe 2023; Design and Verification Conference and Exhibition Europe, Nov. 2023, pp. 7–13.

Hierarchical Framework

1. Read statistical performance model results onto performance objects
2. Inject all parameters in a global PVT set – ensures consistency of sampled value for all VABMs
3. Apply local sensitivity and calculate performance values
4. Feed the calculated performance to functional behavioral blocks



Data Format (.par)

Metadata

Performance data

Parameters data

```
///////////////////////// Metadata /////////////////////////////
// folder: /home/.../simulation/ChargePump
//2024-08-02 13:13:32.289737

//k_cluster: 1
//tau_relevant: 0.01
//tau_lin: 1e-28
//#MC samples: 2000

//////////////////// Performances /////////////////////////////////
moduleName = "cp"
cp_performances = {("i_up_p", 0.000101167012965), ("i_dn_p",-0.00010442588830000001)}

sens_i_up_p = {("Vsupply_unif_1", 3.4996726827876146e-05),("r1_io",
1.0396822042038212e-05),("r19_io", 3.945994105012961e-06),...}
error_mean_i_up_p = -4.724115689876391e-10
error_stddev_i_up_p = 9.192259238706476e-06

sens_i_dn_p = {("Vsupply_unif_1", -3.574452437801205e-05),("r1_io",
-1.1692098714331887e-05),("r2_io", 5.025710131731148e-06),("r19_io", -4.510244056577494e-06),...}
error_mean_i_dn_p = 4.630224785094836e-10
error_stddev_i_dn_p = 9.528540473351679e-06


//////////////////// Parameters //////////////////////////////////////////
parNames = {"Vsupply_unif_1", "r1_io", "r19_io", "r2_io",..}
parMeans = {0.0,...}
parDistType = {"gauss",...}
parStdDev = {1.0,...}
parZscore = {10.0,...}
```

Results – Charge Pump and Loop Filter VABM

Performance metric	Statistical measures	Circuit-level Monte Carlo	VABM	Gaussian Model	FMKL GLD -based model
CP I_{up} [µA]	Mean	99.25	98.90	99.69	99.61
	Std. Dev.	44.9	36.79	29.23	34.08
	Skewness	0.27	0.11	0.63	0.51
	Kurtosis	2.22	2.94	3.21	2.94
	JSD	-	0.0551	0.0433	0.0466
	$\pm 3\sigma$ range	[28.00,210.26]	[-2.51,216.89]	[32.02,228.50]	[27.77,220.99]
CP I_{dn} [µA]	Mean	-101.89	-102.09	-105.29	-105.99
	Std. Dev.	46.18	38.23	32.53	34.08
	Skewness	-0.28	-0.11	-0.48	0.51
	Kurtosis	2.24	2.96	2.87	2.94
	JSD	-	0.0548	0.0425	0.0508
	$\pm 3\sigma$ range	[-219.23,-27.74]	[-227.52,5.11]	[-222.78,-29.95]	[-177.83,15.39]
LF Linear Gain [$\times 10^9$]	Mean	48.74	48.49	49.12	48.89
	Std. Dev.	5.83	4.10	7.25	6.87
	Skewness	0.53	-0.02	0.64	0.47
	Kurtosis	2.97	2.86	2.94	2.96
	JSD	-	0.0039	0.0066	0.0064
	$\pm 3\sigma$ range	[35.09,71.57]	[32.10,70.13]	[33.66,76.41]	[33.89,73.19]
LF P1 [$\times 10^3$ rad s^{-1}]	Mean	525.77	521.16	536.86	528.28
	Std. Dev.	75.13	74.14	106.07	68.71
	Skewness	0.50	0.07	0.67	0.47
	Kurtosis	2.95	2.90	2.89	2.96
	JSD	-	0.0072	0.0097	0.007
	$\pm 3\sigma$ range	[348.48,794.42]	[315.56,782.88]	[317.09,936.14]	[378.22,771.29]
LF Z1 [$\times 10^3$ rad s^{-1}]	Mean	75.32	75.81	75.82	75.70
	Std. Dev.	5.20	6.02	7.17	6.11
	Skewness	0.5	-0.03	0.82	0.65
	Kurtosis	2.94	2.74	2.99	2.62
	JSD	-	0.0059	0.0097	0.002
	$\pm 3\sigma$ range	[63.04,93.91]	[38.70,112.85]	[61.40,107.92]	[67.23,95.40]

Benchmarks - Response Surface Modeling vs Quasi Sensitivity Analysis

		HSPICE MCS	Our proposed PLL behavioral model		Top-down PLL behavioral model	
			Using 1st RSM for behavioral parameters			
		Value	Value	err. (%)	Value	err. (%)
V_{lock} (V)	Mean	0.9941	0.9931	-0.1	0.9254	-6.9
	St. Dev.	0.0343	0.0365	6.4	0.0384	12.0
T_{lock} (us)	Mean	3.3743	3.4489	2.2	3.1641	-6.2
	St. Dev.	0.5764	0.5729	-0.6	0.5447	-5.5
pk-pk Jitter @ 800MHz (ps)	Mean	13.2	12.2	-7.6	8.0	-39.4
	St. Dev.	1.40	1.36	-2.9	4.61	229.3
	Worst	17.0	16.6	-2.4	18.2	7.1
$T_{extraction}$ (hours)		N/A	$4(n+1) \times 1.71 = 34.2$, $n = 4$ (n: # of device parameter variation)			
$T_{simulation}$ (hours)		598.54	2.95		2.10	

		HSPICE MCS	**Our Quasi-SA +BMCS**		Trad. SA +BMCS	
			Using our proposed accurate PLL model			
		Value	Value	err. (%)	Value	err. (%)
V_{lock} (V)	Mean	0.9941	**0.9947**	**0.1**	0.9934	-0.1
	St. Dev.	0.0343	**0.0349**	**1.7**	0.0453	32.1
T_{lock} (us)	Mean	3.3743	**3.4383**	**1.9**	3.4409	2.0
	St. Dev.	0.5764	**0.5717**	**-0.8**	0.5406	-6.2
pk-pk Jitter @ 800MHz (ps)	Mean	13.2	**12.4**	**-6.1**	12.4	-6.1
	St. Dev.	1.40	**1.41**	**0.7**	2.29	63.6
	Worst	17.0	**16.7**	**-1.8**	16.4	-3.5
$T_{extraction}$ (hours)		N/A	**$(n+1) \times 1.71 = 8.55$, $n = 4$ (n: # of device parameter variation)**			
$T_{simulation}$ (hours)		598.54	**3.50**		2.93	

C.-C. Kuo, M.-J. Lee, C.-N. Liu, and C.-J. Huang, “Fast Statistical Analysis of Process Variation Effects Using Accurate PLL Behavioral Models,” *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 56, no. 6, pp. 1160–1172, Jun. 2009, doi: 10.1109/TCSI.2008.2008502.

Benchmarks - Learning-based approaches

- Correlated Bayesian Model Fusion (C-BMF) – Case study Operational Amplifier

Number of PVT parameters	1184 P + 36 VT
Training cost	540 samples
Simulation cost	1.05 Hours
Fitting cost	20 Mins

- Particle Swarm Optimisation (PSO) – Case study PLL

Number of PVT parameters	35
Training cost	200 Monte Carlo Samples
Circuit Monte Carlo	1000 runs over 5 days

[1] Z. Gao, F. Wang, J. Tao, Y. Su, X. Zeng, and X. Li, "Correlated bayesian model fusion: Efficient high-dimensional performance modeling of analog/RF integrated circuits over multiple corners," IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 42, no. 2, pp. 360–370, Feb. 2023, ISSN: 1937-4151. DOI: 10 . 1109/TCAD.2022.3174170.

[2] O. Garitselov, S. P. Mohanty, E. Kougianos, and G. Zheng, "Particle swarm optimization over non-polynomial metamodels for fast process variation resilient design of nano-cmos PLL," in Great Lakes Symposium on VLSI, Salt Lake City, UT, USA, ACM, 2012, pp. 255–258. DOI: 10.1145/2206781.2206843

Jensen-Shannon Divergence (JSD)

KL is the Kullback-Leibler divergence or 'relative entropy'

$$KL(P \,||\, Q) = \sum_i P(i) * log_2\left(\frac{P(i)}{Q(i)}\right)$$

$$KL(P \,||\, Q) = 0 \text{ (P identical to Q)}$$

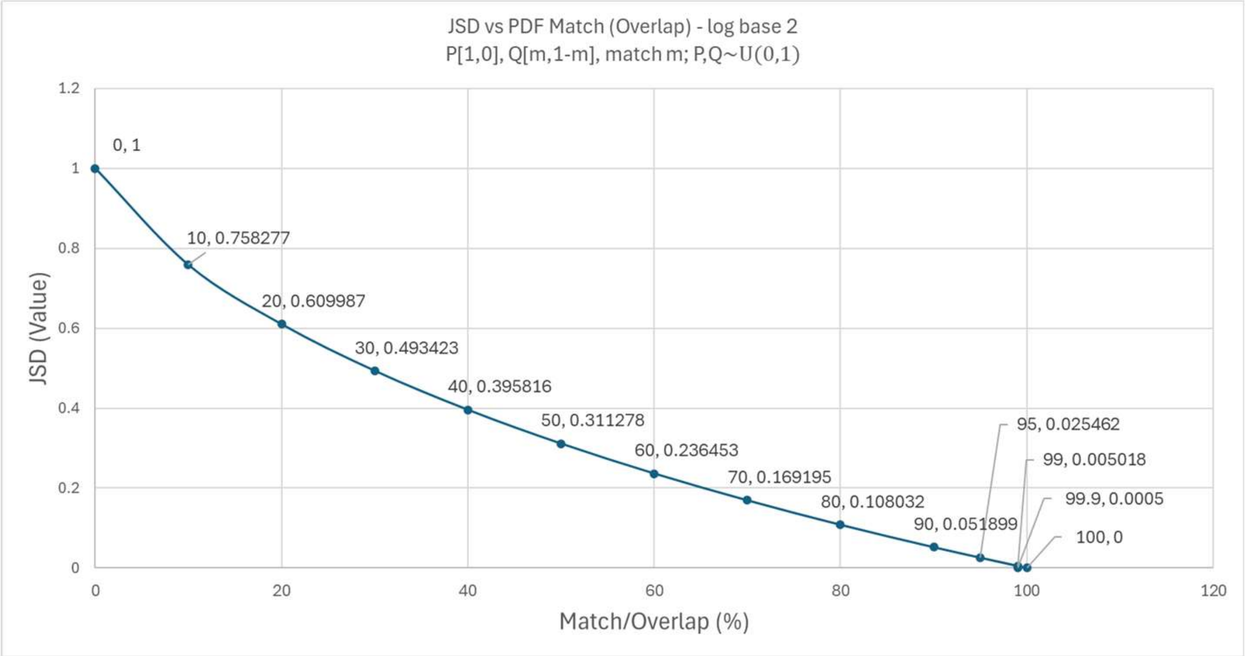
$$KL(P \,||\, Q) \neq KL(Q \,||\, P)$$

JS divergence between two PDFs P and Q

$$JSD(P||Q) = \frac{1}{2}KL(P \,||\, M) + \frac{1}{2}KL(Q \,||\, M)$$

$$M = (P+Q) / 2$$

JSD [0,1], for log base = 2



Jensen-Shannon Divergence
Example: For disjoint distribution

Consider probability distribution functions(PDFs) of P and Q, where

$$P(X = x) = \frac{P(number\ of\ occurances\ of\ x)}{Total\ number\ of\ elements}$$

P = [1,0,0]
Q = [0,0,1]

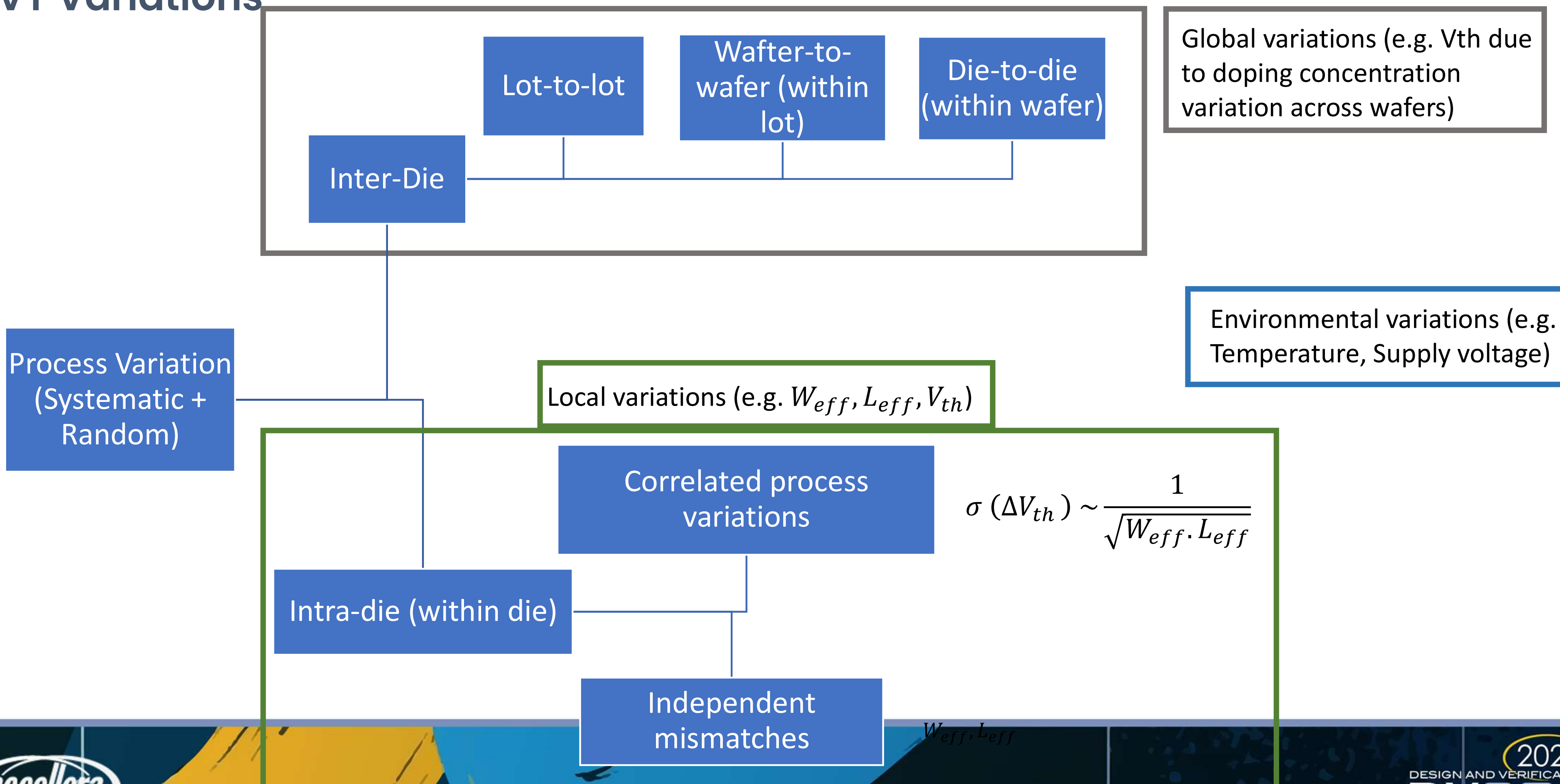
then $M = \frac{1}{2}(P + Q) = \frac{1}{2}([1, 0, 0] + [0, 0, 1]) = [0.5, 0, 0.5]$

$$\frac{1}{2}\mathrm{KL}(P||M) = \frac{1}{2}\left[1 \log\left(\frac{1}{0.5}\right) + 0 \log\left(\frac{0}{0}\right) + 0 \log\left(\frac{0}{0.5}\right)\right] = \frac{1}{2}\log_2 2$$

$$\frac{1}{2}\mathrm{KL}(Q||M) = 0 + 0 + \frac{1}{2}\left[1 \log\left(\frac{1}{0.5}\right)\right] = \frac{1}{2}\log_2 2$$

$JSD(P||Q) = \frac{1}{2}\mathrm{KL}(P||M) + \frac{1}{2}\mathrm{KL}(Q||M) = 1$

PVT Variations



X. Li, J. Le, and L. T. Pileggi, “Statistical performance modeling and optimization,” vol. 1, no. 4, pp. 335–351.

Functional Behavioral Modeling

Analog behavioral models are typically constructed using hardware description languages (HDLs), such as SystemC-AMS and Verilog-AMS.

- Use of equation-based approaches to model circuit behavior
- significantly faster simulation compared to SPICE
- may lack accuracy because of various simplifications
- application-specific, significant efforts required for adaptation

[2] G. Zheng, S. P. Mohanty, E. Kougianos, and O. Garitselov, 'Verilog-AMS-PAM: verilog-AMS integrated with parasitic-aware metamodels for ultra-fast and layout-accurate mixed-signal design exploration', in Proceedings of the great lakes symposium on VLSI, 2012, pp. 351–356.

[3] M. D. Hershenson, S. P. Boyd, and T. H. Lee, 'Optimal design of a CMOS op-amp via geometric programming', IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems, vol. 20, no. 1, pp. 1–21, Jan. 2001.

[4] G. Stehr, M. Pronath, F. Schenkel, H. Graeb, and K. Antreich, 'Initial sizing of analog integrated circuits by centering within topology-given implicit specifications', in ICCAD-2003. International Conference on Computer Aided Design (IEEE Cat. No.03CH37486), 2003, pp. 241–246

[5] H. Tang, 'Symbolic Statistical Analysis of SNR Variation for Delta–Sigma Modulators', IEEE Transactions on Circuits and Systems II: Express Briefs, vol. 54, no. 8, pp. 720–724, Aug. 2007.